The plumbing challenges of hybrid modelling







The plumbing challenges of hybrid modelling



Interfacing ocean models with DL frameworks (1/3)



stable, robust, low abstraction languages



high abstraction, fast evolving languages







runs only on CPUs





cloud ready

natively runs on GPUs



Ocean circulation models







Interfacing ocean models with DL frameworks (3/3)



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- OASIS : exchange of 3D data between different codes
- Eophis : simplified deployment of ML models w/ OASIS
- Requires some change to the NEMO code
- Key : portability, domain decomposition

https://github.com/meom-group/eophis





Interfacing ocean models with DL frameworks (3/3)



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https://github.com/meom-group/eophis





The (real) challenge of online training (1/2) online learning

offline learning





from pre-existing data

Online training improves performance, stability, generalisation Frezat et al. 2022; List et al. 2024



 $\partial_t \mathbf{y} + G(\mathbf{y}) + \mathcal{M}_{NN}(\mathbf{y}) = f$

along a trajectory

(a.k.a : a posteriori, solver-in-the-loop, end-to-end, auto-regressive roll-outs)



The (real) challenge of online training (2/2)



For time evolving problems, with

$$\mathbf{y}(t + \Delta t) = E_m \circ \cdots \circ E_1(\mathbf{y}(t))$$

The gradient of the loss involves

$$\frac{\partial \mathscr{M}}{\partial \theta} \stackrel{}{=} \frac{\partial E}{\partial \theta} \stackrel{}{=} \frac{\partial (E_m \circ \cdots \circ E_1)}{\partial \theta}$$

 $\frac{\partial \mathscr{L}}{\partial \theta} (\mathbf{Z}, \mathscr{M}(\mathbf{y} \mid \theta)) = \frac{\partial \mathscr{M}}{\partial \theta} (\mathbf{y} \mid \theta) \frac{\partial \mathscr{L}}{\partial \mathscr{M}}$

gradient of the loss

 $\mathcal{M} \equiv E$ Auto-regressive operator (time)

> tricky without Automatic Differenciation (AD) !

 $\partial E_m \quad \partial E_2 \ \partial E_1$ $\frac{\partial E_{m-1}}{\partial E_{1}} \frac{\partial E_{1}}{\partial \theta}$





The challenge of online training strategies (2/2)



For time evolving problems, with

$$\mathbf{y}(t + \Delta t) = E_m \circ \cdots \circ E_1(\mathbf{y}(t))$$

The gradient of the loss involves

$$\frac{\partial \mathscr{M}}{\partial \theta} \stackrel{}{=} \frac{\partial E}{\partial \theta} \stackrel{}{=} \frac{\partial (E_m \circ \cdots \circ E_1)}{\partial \theta}$$

AD is readily available in some language





 $\mathcal{M} \equiv E$ temporal evolution operator

> tricky without Automatic Differenciation (AD) !

 $\partial E_m \quad \partial E_2 \ \partial E_1$ $\frac{\partial E_{m-1}}{\partial E_1} \frac{\partial E_1}{\partial \theta}$





The challenge of online training strategies (2/2)





See eg Sapienza et al. 2024 https://arxiv.org/abs/2406.09699

AD is readily available in some language





Supervised or residual loss L



But AD used yet in climate models...

Differentiable programming

- programs composed of differentiable building blocks - building blocks : trainable and procedural code components - trainable end-to-end with gradient based optimisation











Al-native hybrid geoscientific models

Article Neural general circulation models for weather and climate

https://doi.org/10.1038/s41586-024-07744-y Dmitrii Kochkov¹⁸⁵⁵, Janni Yuval¹⁸⁵⁵, Ian Langmore¹⁸, Peter Norgaard¹⁸, Jamie Smith¹⁸ Griffin Mooers', Milan Klöwer', James Lottes', Stephan Rasp', Peter Düben', Sam Hatfield', Received: 13 November 2023 Peter Battaglia⁴, Alvaro Sanchez-Gonzalez⁴, Matthew Willson⁴, Michael P. Brenner¹⁵ & Accepted: 15 June 2024 Stephan Hover¹⁶¹ Published online: 22 July 2024 General circulation models (GCMs) are the foundation of weather and climate Open access prediction¹³. GCMs are physics-based simulators that combine a numerical solver Check for updates for large-scale dynamics with tuned representations for small-scale processes such as cloud formation. Recently, machine-learning models trained on reanalysis data have achieved comparable or better skill than GCMs for deterministic weather forecasting34. However, these models have not demonstrated improved ensemble forecasts, or shown sufficient stability for long-term weather and climate simulations. Here we present a GCM that combines a differentiable solver for atmospheric dynamics with machine-learning components and show that it can generate forecasts of deterministic weather, ensemble weather and climate on par with the best machine-learning and physics-based methods. NeuralGCM is competitive with machine-learning models for one- to ten-day forecasts, and with the European Centre for Medium-Range Weather Forecasts ensemble prediction for one- to fifteen-day forecasts. With prescribed sea surface temperature, NeuralGCM can accurately track climate metrics for multiple decades, and climate forecasts with 140-kilometre resolution show emergent phenomena such as realistic frequency and trajectories of tropical cyclones. For both weather and climate, our approach offers orders of magnitude computational savings over conventional GCMs, although our model does not extrapolate to substantially different future climates. Our results show that end-to-end deep learning is compatible with tasks performed by conventional GCMs and can enhance the large-scale physical simulations that are essential for understanding and predicting the Earth system. Solving the equations for Earth's atmosphere with general circula- demonstrating state-of-the-art deterministic forecasts for 1- to 10-day weather prediction at a fraction of the computational cost of traditional tion models (GCMs) is the basis of weather and climate prediction¹². Over the past 70 years, GCMs have been steadily improved with better models14. Machine-learning atmospheric models also require considernumerical methods and more detailed physical models, while exploit- ably less code, for example GraphCast³ has 5,417 lines versus 376,578 ing faster computers to run at higher resolution. Inside GCMs, the lines for the National Oceanic and Atmospheric Administration's FV3 unresolved physical processes such as clouds, radiation and precipi- atmospheric model19 (see Supplementary Information section A for tation are represented by semi-empirical parameterizations. Tuning details). GCMs to match historical data remains a manual process³, and GCMs Nevertheless, machine-learning approaches have noteworthy retain many persistent errors and biases*⁸. The difficulty of reducing limitations compared with GCMs. Existing machine-learning models uncertainty in long-term climate projections' and estimating distribu- have focused on deterministic prediction, and surpass deterministic tions of extreme weather events³⁰ presents major challenges for climate unmerical weather prediction in terms of the aggregate metrics for mitigation and adaptation¹¹. which they are trained^{3,4}. However, they do not produce calibrated Recent advances in machine learning have presented an alter-uncertainty estimates⁴, which is essential for useful weather forecasts¹. native for weather forecasting 14.11.13. These models rely solely on Deterministic machine-learning models using a mean-squared-error machine-learning techniques, using roughly 40 years of historical loss are rewarded for averaging over uncertainty, producing unrealisdata from the European Center for Medium-Range Weather Forecasts tically blurry predictions when optimized for multi-day forecasts³³¹. (ECMWF) reanalysis v5 (ERA5)²⁴ for model training and forecast initiali-Unlike physical models, machine-learning models misrepresent derived zation. Machine-learning methods have been remarkably successful. (diagnostic) variables such as geostrophic windth. Furthermore,

Google Research, Mountain View, CA, USA, "Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA, USA, "European Centre for Medium-Range Weather Forecasts, Reading, UK. "Google DeepMind, London, UK. "School of Engineering and Applied Sciences, Harvard Drivensity, Cambridge, MA, USA. "These authors contributed equally Dmbni Kochkov, Janni Yuval, tan Langmone, Peter Norgaand, Jamie Smith, Stephan Hoyer. 🧤 mail: dkochkov@google.com; janniyuval@google.com; shoyer@google.com

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https://doi.org/10.1038/s41586-024-07744-y

Kochkov et al. (2024)

(a)

Inputs

Outputs





https://github.com/google-research/dinosaur https://github.com/google-research/neuralgcm





Towards Al-native hybrid climate models ?



5.



Towards Al-native hybrid climate models ?





Al-native hybrid climate models ?

Earth System Observation Data

Ground truth for the validation of process-based models

Physical Equation-driven Earth and Climate Modelling

Main tool for quantifying the Earth's state under ongoing anthropogenic forcing

Contains persistent error sources

Process-based models and neural networks will be coupled as actively learning hybrid models

Irrgang et al. (2021)

Successive research on explainable AI will make hybrid models more physically interpretable Combining the advantages of process-based with machine learning models

Neural Earth System Modelling

Available data pool for neural network training environments

> Earth Data-driven Machine Learning

Highly specialized agents that uncover hidden patterns and geophysical quantities

Lack of process knowledge

Hybrid models start to outperform the predictive power of traditional models

Betting harnessing observations & hi-fidelity **simulations**

... for optimising

- model parameters
- numerical schemes
- subgrid closures

Differentiable programming in earth system models ?



Ocean circulation models













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Veros 1.5.1+51.g4039f76.dirty documentation

Versatile Ocean Simulation in Pure Python

Veros, *the versatile ocean simulator*, aims to be the swiss army knife of ocean modeling. It is a fullfledged primitive equation ocean model that supports anything between idealized toy models and realistic, high-resolution, global ocean simulations. And because Veros is written in pure Python, the days of struggling with complicated model setup workflows, ancient programming environments, and obscure legacy code are finally over.

In a nutshell, we want to enable high-performance ocean modelling with a clear focus on flexibility and usability.

Veros supports a NumPy backend for small-scale problems, and a high-performance JAX backend with CPU and GPU support. It is fully parallelized via MPI and supports distributed execution on any number of nodes, including multi-GPU architectures (see also our benchmarks).

The dynamical core of Veros is based on pyOM2, an ocean model with a Fortran backend and Fortran and Python frontends.

If you want to learn more about the background and capabilities of Veros, you should check out A short introduction to Veros. If you are already convinced, you can jump right into action, and learn how to get started instead!



Modern code and compute : simple to write, scales, runs on any hardware

Atmos

Ocean



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④ ① + □ ••• D veros.readthedocs.io/en/latest/ Ξ Veros 1.5.1+51.g4039f76.dirty documentation .ö. ⊡ 0 Versatile Ocean Simulation in Pure Python Veros, the versatile ocean simulator, aims to be the swiss army knife of ocean modeling. It is a fullfledged primitive equation ocean model that supports anything between idealized toy models and realistic, high-resolution, global ocean simulations. And because Veros is written in pure Python, the days of struggling with complicated model setup workflows, ancient programming environments, and obscure legacy code are finally over. In a nutshell, we want to enable high-performance ocean modelling with a clear focus on flexibility and usability. Veros supports a NumPy backend for small-scale problems, and a high-performance JAX backend with CPU and GPU support. It is fully parallelized via MPI and supports distributed execution on any number of nodes, including multi-GPU architectures (see also our benchmarks). The dynamical core of Veros is based on pyOM2, an ocean model with a Fortran backend and Fortran and Python frontends. If you want to learn more about the background and capabilities of Veros, you should check out A short introduction to Veros. If you are already convinced, you can jump right into action, and learn how to get started instead! ... because the Baroque is over. See also We outline some of our design philosophy and current direction in this blog post. START HERE A short introduction to Veros The vision Features Getting started Installation Setting up a model Running Veros Enhancing Veros Advanced installation Using JAX









□ README MIT license 🖉 🗄	Deployments 408	
Oceananigans.jl	github-pages 5 years ago + 407 deployments	
E Fast and friendly ocean-flavored Julia software for simulating incompressible fluid dynamics in	Languages	
Cartesian and spherical shell domains on CPUs and GPUs.	Julia 97.9% Mathematica 1.6%	
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Buildkite CPU+GPU		
Oceananigans is a fast, friendly, flexible software package for finite volume simulations of the nonhydrostatic and hydrostatic Boussinesq equations on CPUs and GPUs. It runs on GPUs (wow, fast!), though we believe Oceananigans makes the biggest waves with its ultra-flexible user interface that makes simple simulations easy, and complex, creative simulations possible.		
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Contents		
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Installation instructions		
Running your first model		
The Oceananigans knowledge base		
• Citing		
Contributing		
Movies Deep convection		
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Winds blowing over the ocean		
 Free convection with wind stress 		
Performance benchmarks		
Installation instructions		
Oceananigans is a registered Julia package. So to install it.		
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Allowing a seamless integration with Al



stable, robust, low abstraction languages



high abstraction, fast evolving languages

Allowing a seamless integration with Al

Data-driven modelling





high abstraction, fast evolving languages

Physics-based modelling





high abstraction, fast evolving languages





Neural general circulation models for weather and climate

https://doi.org/10.1038/s41586-024-07744-y Dmitrii Kochkov¹⁸⁵⁵, Janni Yuval¹⁸⁵⁵, Ian Langmore¹⁸, Peter Norgaard¹⁸, Jamie Smith¹⁸ Griffin Mooers', Milan Klöwer', James Lottes', Stephan Rasp', Peter Düben', Sam Hatfield', Received: 13 November 2023 Peter Battaglia⁴, Alvaro Sanchez-Gonzalez⁴, Matthew Willson⁴, Michael P. Brenner¹⁵ & Accepted: 15 June 2024 Stephan Hover¹⁶¹ Published online: 22 July 2024 General circulation models (GCMs) are the foundation of weather and climate Open access prediction¹³. GCMs are physics-based simulators that combine a numerical solver Check for updates for large-scale dynamics with tuned representations for small-scale processes such as cloud formation. Recently, machine-learning models trained on reanalysis data have achieved comparable or better skill than GCMs for deterministic weather forecasting34. However, these models have not demonstrated improved ensemble forecasts, or shown sufficient stability for long-term weather and climate simulations. 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Furthermore, Google Research, Mountain View, CA, USA, "Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA, USA, "European Centre for Medium-Range Weather Forecasts, Reading, UK. "Google DeepMind, London, UK. "School of Engineering and Applied Sciences, Harvard Drivensity, Cambridge, MA, USA. "These authors contributed equally Dmbni Kochkov, Janni Yuval, tan Langmone, Peter Norgaard, Jamie Smith, Stephan Hoyer. 🤏 mail: dkochkov@google.com; janniyuval@google.com; shoyer@google.com

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(a)

Inputs

Outputs



Kochkov et al. (2024)



https://github.com/google-research/dinosaur https://github.com/google-research/neuralgcm



Al-native hybrid climate models ?

Kochkov et al. (2024)

https://arxiv.org/abs/2311.07222



ERA5

Hybrid w/ online : non-blurry forecast + stable simulators (runs ~10 years)

Total column water, 0-15 days



NeuralGCM





Al-native hybrid climate models ?

See P. Gentine's keynote during

TRACCS General Assembly





Earth Observations

Km-scale climate models

LES

DNS

LEGO land :

Versatile models allowing exploration of advanced ML workflows

> Harnessing global observations !

Hybrid (physics+ML) ESMs



Eyring, Gentine, et al., 2024 <u>https://doi.org/10.1038/s41561-024-01527-w</u>

Differentiable programming in earth system models







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- Described how this can be done in practice today
- Further progress require deeper recast of our models
- Exploration w/ new generation of Al-native hybrid models

Integrating model-based - Exciting time for cross-disciplinary investigations !





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Summary

Why we are augmenting ocean models w/ trained comp.











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